Figure B.7

Algebra Unit Before UbD

Topic
Associative, Commutative, and Distributive Properties; Order of Operations
Activities
1. Direct instruction on the order of operations (PEMDAS): parentheses, exponents, multiplication, division, addition, subtraction. 2. In-class exercises on the order of operations. 3. Direct instruction on the associative and commutative properties: $(a + b) + c = a + (b + c)$; $ab = b \times a$ 4. In-class and online exercises on the associative and commutative properties: simplify $-12x - 5x + 3a + x$. Justify each step. 5. Direct instruction, in class and online exercises, on the distributive property: $a(b + c) = ab + ac$; simplify $2(4x + y) - 2x$ 6. Review chapter in preparation for test.
Assessments
1. Quiz on associative property, commutative property, and distributive property. For example, name the property $a + b + 2 = a + 2 + b$; write the product using the distributive property: 6($5.95) = 2. Quiz on order of operations: $(3 + 4)\ 6 - 12^2 + 4$ 3. Chapter test on properties and rules. 4. Homework problems on the properties and rules.

Figure B.8

Algebra Unit

<table>
<tr><th colspan="3">Stage 1—Desired Results</th></tr>
<tr><td rowspan="8">Established Goals

Common Core Math Standards

Interpret the structure of expressions
1. Interpret expressions that represent a quantity in terms of its context.

Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Rewrite rational expressions
6. Rewrite simple rational expressions in different forms.

Mathematical practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.</td><td colspan="2">Transfer</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning to…

• Solve problems by simplifying them, using equivalent statements based on the properties of real numbers and the order of operations.
• Analyze when any rule in any system (language, law, math) is an essential principle or merely conventional.</td></tr>
<tr><td colspan="2">Meaning</td></tr>
<tr><td>UNDERSTANDINGS
Students will understand that…

1. Mathematics is a language, and over the centuries mathematicians have come to agree on certain conventions, or ways of doing things, so that we can communicate our intentions clearly and efficiently.

2. In mathematics, we accept certain truths as necessary to permit us to solve problems with logical certainty (e.g., the properties of real numbers), while other rules are conventions that we assume just for effective communication.

3. We can use the commutative, associative, and distributive properties to turn complex and unfamiliar expressions into simpler and familiar ones to solve problems.</td><td>ESSENTIAL QUESTIONS
Students will keep considering…

1. Why and when is it important to come to agreement on procedural rules (in mathematics, sports/games, language)?

2. What important rules and conventions are required to make algebra “work”? How can we distinguish between essential properties and agreed-upon, but arbitrary, conventions?

3. Why and how do we simplify algebraic expressions?</td></tr>
<tr><td colspan="2">Acquisition of Knowledge and Skill</td></tr>
<tr><td>Students will know…

1. The commutative property and to which operation it applies (and when it does not apply).

2. The associative property and to which operation it applies (and when it does not apply).</td><td>Students will be skilled at…

1. Recognizing and applying the commutative, associative, and distributive properties to simplify algebraic expressions.

2. Using the convention of "order of operations" to perform calculations and simplify algebraic expressions.</td></tr>
</table>

Figure B.8

Algebra Unit *(continued)*

		Acquisition of Knowledge and Skill (*continued*)	
		Students will know…	*Students will be skilled at…*
		3. The distributive property and to which operation it applies (and when it does not apply). 4. The "order of operations" mathematicians use and why it is needed. 5. What PEMDAS means. 6. What it means to simplify an expression.	3. Recognizing situations where properties do not apply or are optional. 4. Identifying equivalence that results from properties and equivalence that is the result of computation. 5. Justifying steps in a simplification or computation by citing applicable laws, properties, and conventions.
Stage 2—Evidence			
Code*	**Evaluative Criteria**		
All ST 1	Well argued, accurate, creative, effective	PERFORMANCE TASK(S): Students will show that they really understand by evidence of… 1. Their ability to realize and apply the difference between (a) necessary logical implications and (b) arbitrary but needed conventions. 2. Showing that they see why PEMDAS is a convention while the associative, commutative, and distributive properties are logical foundations. 3. Using their understanding of PEMDAS and each property to solve problems and explain why the answers and steps are correct. Tasks might include:	
U, EQ	Clear, thoughtful, supported	• PR Campaign for the Rules. Mathematical rules have gotten a bad rap—they confuse people, are used to torture math students, and are too complicated (say critics). You believe that the rules are logical, necessary, and not so mysterious when you really understand them. (The real blame lies not on the rules but on people who force students to memorize rules they don't really understand.) To help remedy this unfortunate situation, you have been hired by the National Mathematics Education Association to design advertisements that explain to peers and younger students why math rules and properties work—and what would happen if we did not have them. Use practical and interesting real-world examples to illustrate your points. Use print, graphics (e.g., poster, website), music (song or rap), or video (e.g., iMovie) to sell your ideas. • Algebra Study Guide. Create a portion of a study guide to help future algebra students understand the associative, commutative, and distributive properties. Make up an original, real-world problem that involves one or more of the properties. Include a detailed explanation of the mathematical procedures and reasoning to explain how each property is used to simplify and solve the problem.	

Figure B.8

Algebra Unit *(continued)*

<table>
<tr><th colspan="3">Stage 2—Evidence (continued)</th></tr>
<tr><th>Code*</th><th>Evaluative Criteria</th><th></th></tr>
<tr><td>U, EQ</td><td>Thorough, thoughtful, organized</td><td>OTHER EVIDENCE:

Students will show they have achieved Stage 1 goals by…
• Providing written or oral response to one of the first three essential questions.</td></tr>
<tr><td>All K, S</td><td>Sufficient independent understanding of PEMDAS and key properties</td><td>• Developing a journal of Rules for Success. The students will keep an ongoing journal all year of accumulating insight about which rules and properties will lead to success in the study of algebra. Include examples that show the rule or property correctly applied, as well as common mistakes. Answer the question “Why can’t you divide by zero? Is that a convention or property?”
• Passing all quizzes from the textbook on basic properties and order of operations (as in the original unit). Students should also troubleshoot examples and explain misapplications of the convention or property, and—at the end—explain the difference between a property and a convention.</td></tr>
<tr><th colspan="3">Stage 3—Learning Plan</th></tr>
<tr><th>Code*</th><th colspan="2">Pre-assessment
Pre-assess: Give an ungraded quiz using simple numbers but with tricky order-of-operations decisions. Although the problems appear easy, students disagree about the answers. Discuss the different answers without stating the rule. As students describe how they got their answers, pose questions such as “Why is that approach OK? Why is that not OK? Couldn’t we all agree to add before we multiply—or work from right to left? Should we?” Work through one simple example together to get the discussion started; e.g., 5 – 3 x 2.</th></tr>
<tr><td></td><td>Learning Events

Student success at transfer, meaning, and acquisition depends upon…
Often, students are given properties and conventions, told to memorize them, and then drilled on their use. The following inquiry activities will engage students in coming to understand the difference between properties and conventions—in algebra and beyond.</td><td>Progress Monitoring
Predictable rough spots in their learning that need constant monitoring:
• Forgetting the mnemonic and its meaning
• Not grasping why PEMDAS is needed yet conventional</td></tr>
</table>

Figure B.8

Algebra Unit *(continued)*

Stage 3—Learning Plan (***continued***)		
	Learning Events (*continued*)	*Progress Monitoring—(continued)*
M, T Hook	Find value and interest in the study of rules, laws, and conventions. For example, begin the unit with an exploration of rules of games. Share with students a few little-known but interesting rule changes (e.g., foul third strike equals out in baseball; zone defense legal then oulawed in NBA. See http://www.baseball-almanac.com/rulechng.shtml and http://www.nba.com/analysis/rules_history.html). Assign homework on research into their favorites. Pose questions such as "Given the rules of your favorite game, which rules are essential? Which rules are just conventions that could be changed without fundamentally messing up the game?" The goal is to help students come to the understanding that "some rules are essential to the game, while others are enacted but could be changed" (e.g., the three-point line in basketball).	• Not understanding that conventions differ from core and logically derived principles
M, A	Tell students that you have declared the next 10 minutes as "Do Your Own Thing" time. You're going to suspend ALL the rules of math as we know them; every answer is a potential good answer. Distribute problems involving order of operations and properties, and encourage the students to work in teams to come up with as many plausible answers as possible. Then, discuss the various "answers." (The goal is to help students come to the understanding that rules and conventions are necessary for accuracy in mathematics.)	Some groups may have difficulty generating plausible alternatives to the "same" problem. Some groups may have difficulty drawing generalizations from the answers. Be ready with prompting questions to help them make meaning with minimal assistance.
M, A	Give students a new set of problems that will lead to discussion of the main properties (commutative, associative, and distributive) without using those names. It's valuable to use more complex examples with many possible answers, but also include some simple examples, like 5 x 3 + 4 and 5 – 3 x 4, to illustrate properties and when they are true. Lead a discussion of possible answers for each exercise. Be sure that students have had time to explore the problems fully and discuss them in pairs or small groups.	At some point use an ungraded writing prompt—"Why can we multiply by zero but not divide by zero? Do you think that is a convention or the result of a fundamental property?"—to check for their understanding of the properties.
A, M	Know what a law is, versus a convention, via direct instruction. Introduce the idea that some of the things students learn in mathematics are not natural truths but agreed-upon human conventions. For example, order of operations does not matter for addition. However, some properties are essential because we don't want 1 + 1 to equal more than one answer. Introduce PEMDAS and give students practice in using this mnemonic for order of operations. Then, try to get students to infer the importance of the three core properties—and any others they think of as important to make math "work." After exploring their ideas, introduce the three properties from the textbook.	

*Codes: A = Acquisition, EQ = Essential Question, K = Knowledge, M = Meaning Making, S = Skill, ST = Standard, T = Transfer, U = Understanding